Maths EE - Common Errors

10 March, 2010
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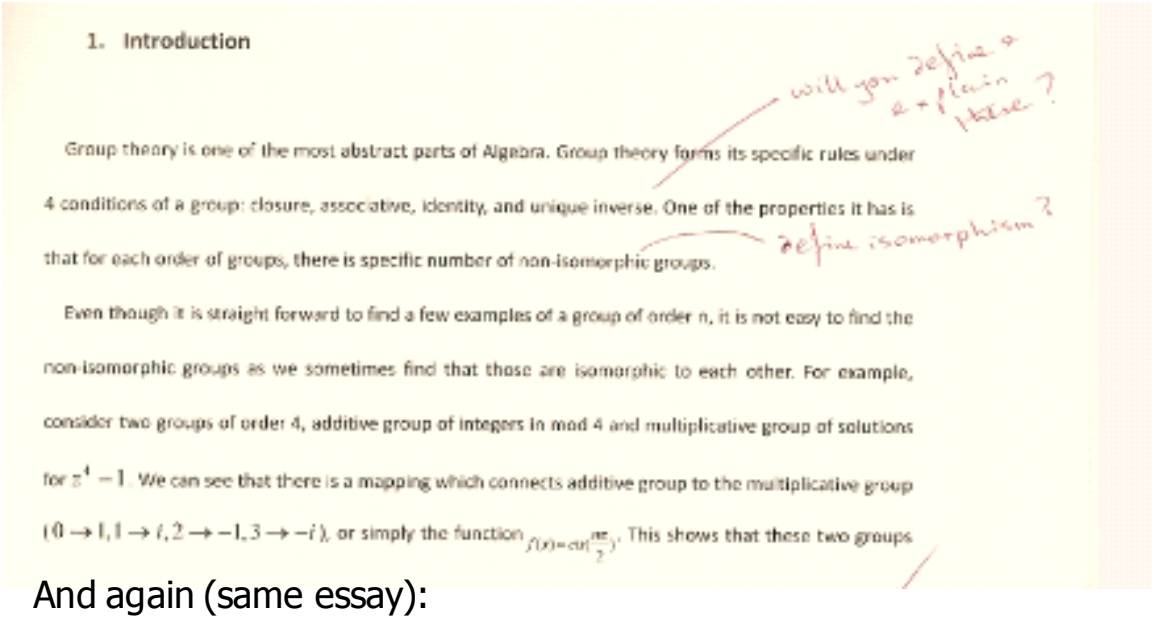
1. **Make sure students follow the EE Assessment Criteria. Students should be shown the criteria, and all should clearly:**

- state their research question (2 marks)
- give an introduction (2 marks)
- give a conclusion (2 marks)
- Make sure the formal elements of title page, table of contents, page numbers, 4000 words max, bibliography etc are present (4 marks)
- Precede the essay with an abstract 300 words max (2 marks)

Thus 12 marks are possible without doing any maths if the basic rules are followed.

2. **Make sure definitions are fully explained. If a theorem is used whose proof is too difficult, it should at least be explained by a clear example. Students need to demonstrate that they fully understand what they are doing.**

e.g. in a good essay looking at non-isomorhic groups of small order:



1. Introduction

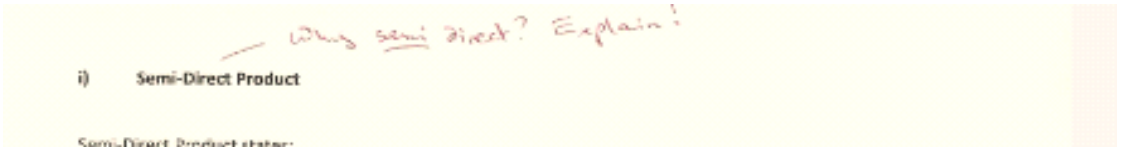
will you define & explain these?

Group theory is one of the most abstract parts of Algebra. Group theory forms its specific rules under 4 conditions of a group: closure, associative, identity, and unique inverse. One of the properties it has is that for each order of groups, there is specific number of non-isomorphic groups.

define isomorphism?

Even though it is straight forward to find a few examples of a group of order n, it is not easy to find the non-isomorphic groups as we sometimes find that those are isomorphic to each other. For example, consider two groups of order 4, additive group of integers in mod 4 and multiplicative group of solutions for $z^4 - 1$. We can see that there is a mapping which connects additive group to the multiplicative group $(0 \to 1, 1 \to i, 2 \to -1, 3 \to -i)$, or simply the function $f(x) = \cos(\frac{\pi x}{2})$. This shows that these two groups

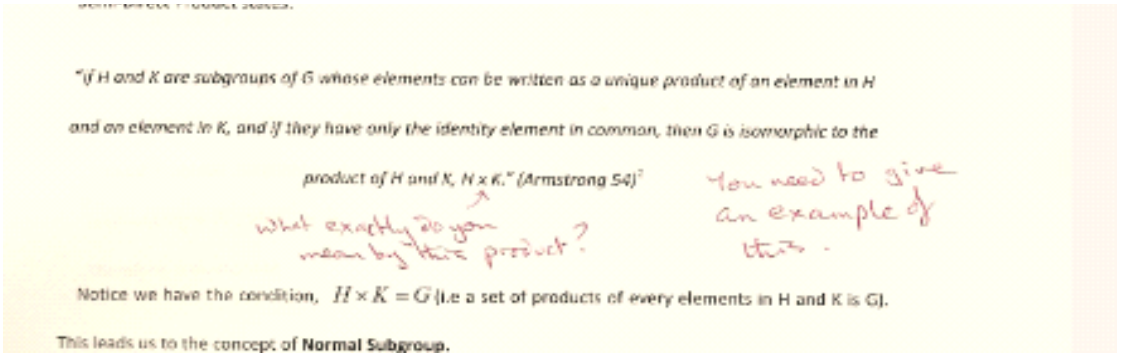
And again (same essay):



Why semi direct? Explain!

i) **Semi-Direct Product**

Semi-Direct Product states:



"if H and K are subgroups of G whose elements can be written as a unique product of an element in H and an element in K, and if they have only the identity element in common, then G is isomorphic to the product of H and K, $H \times K$." (Armstrong 54)

What exactly do you mean by this product?

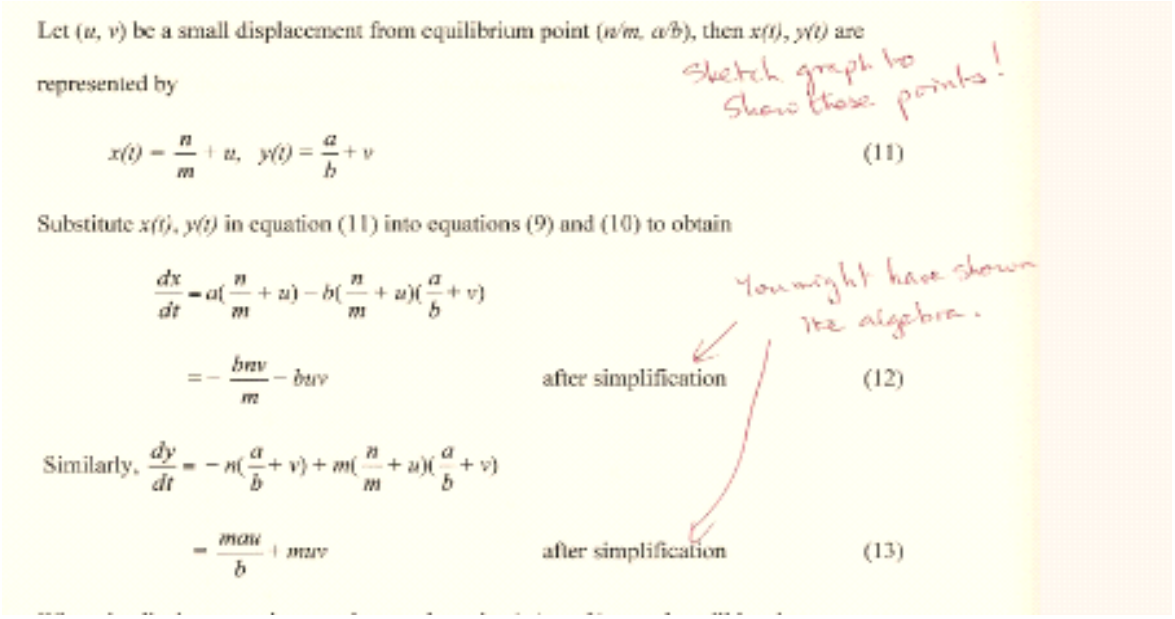
You need to give an example of this.

Notice we have the condition, $H \times K = G$ (i.e a set of products of every elements in H and K is G).

This leads us to the concept of **Normal Subgroup.**

In fact, this became a very good essay (30ish) as the candidate found an excellent way of listing all the non-isomorphic groups of groups of order up to 15, but he would have scored higher had his explanations been clearer. (Categories D to G likely to be 3 rather than 4)

3. Show the steps in the algebra. If steps are missed - has the student taken this from a source (if so, it should be cited)? - does the student really understand what is going on?



Let (u, v) be a small displacement from equilibrium point $(n/m, a/b)$, then $x(t)$, $y(t)$ are represented by

Sketch graph to show these points!

$$x(t) = \frac{n}{m} + u, \quad y(t) = \frac{a}{b} + v \qquad (11)$$

Substitute $x(t)$, $y(t)$ in equation (11) into equations (9) and (10) to obtain

$$\frac{dx}{dt} = a\left(\frac{n}{m} + u\right) - b\left(\frac{n}{m} + u\right)\left(\frac{a}{b} + v\right)$$

$$= -\frac{bnv}{m} - buv \qquad \text{after simplification} \qquad (12)$$

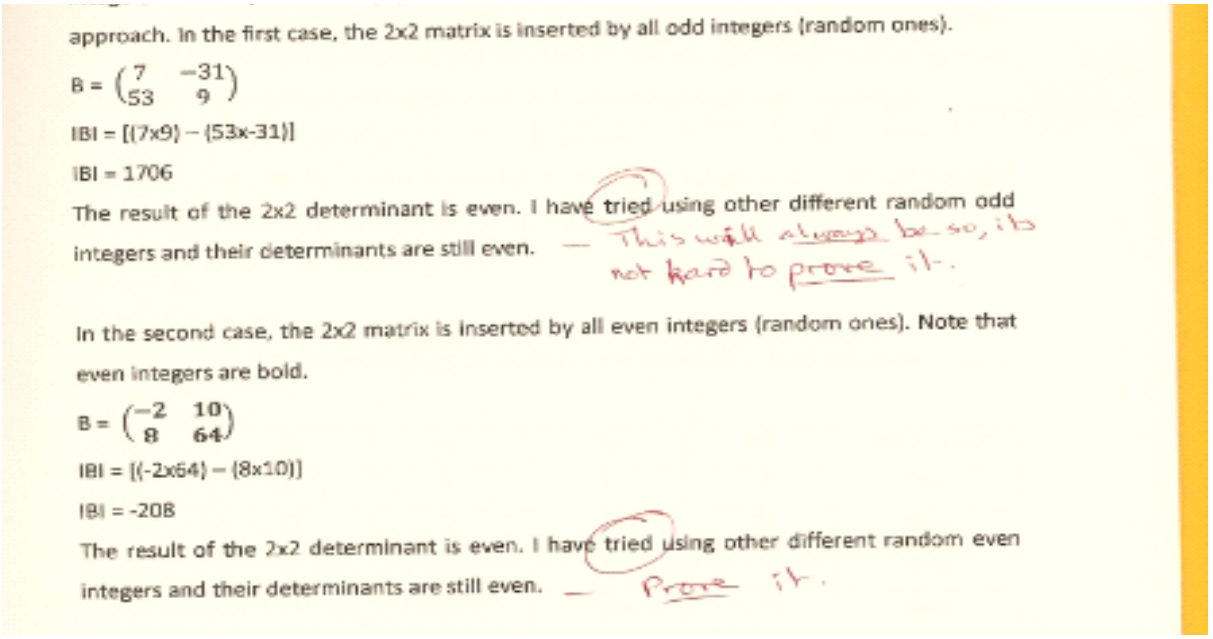
You might have shown the algebra.

$$\text{Similarly, } \frac{dy}{dt} = -n\left(\frac{a}{b} + v\right) + m\left(\frac{n}{m} + u\right)\left(\frac{a}{b} + v\right)$$

$$= \frac{mau}{b} + mav \qquad \text{after simplification} \qquad (13)$$

This is likely to lose marks in categories E and F

4. Encourage students to prove conjectures that can readily be proved.



approach. In the first case, the 2x2 matrix is inserted by all odd integers (random ones).

$B = \begin{pmatrix} 7 & -31 \\ 53 & 9 \end{pmatrix}$

IBI = [(7x9) – (53x-31)]

IBI = 1706

The result of the 2x2 determinant is even. I have tried using other different random odd integers and their determinants are still even. — This will always be so, its not hard to prove it.

In the second case, the 2x2 matrix is inserted by all even integers (random ones). Note that even integers are bold.

$B = \begin{pmatrix} \mathbf{-2} & \mathbf{10} \\ \mathbf{8} & \mathbf{64} \end{pmatrix}$

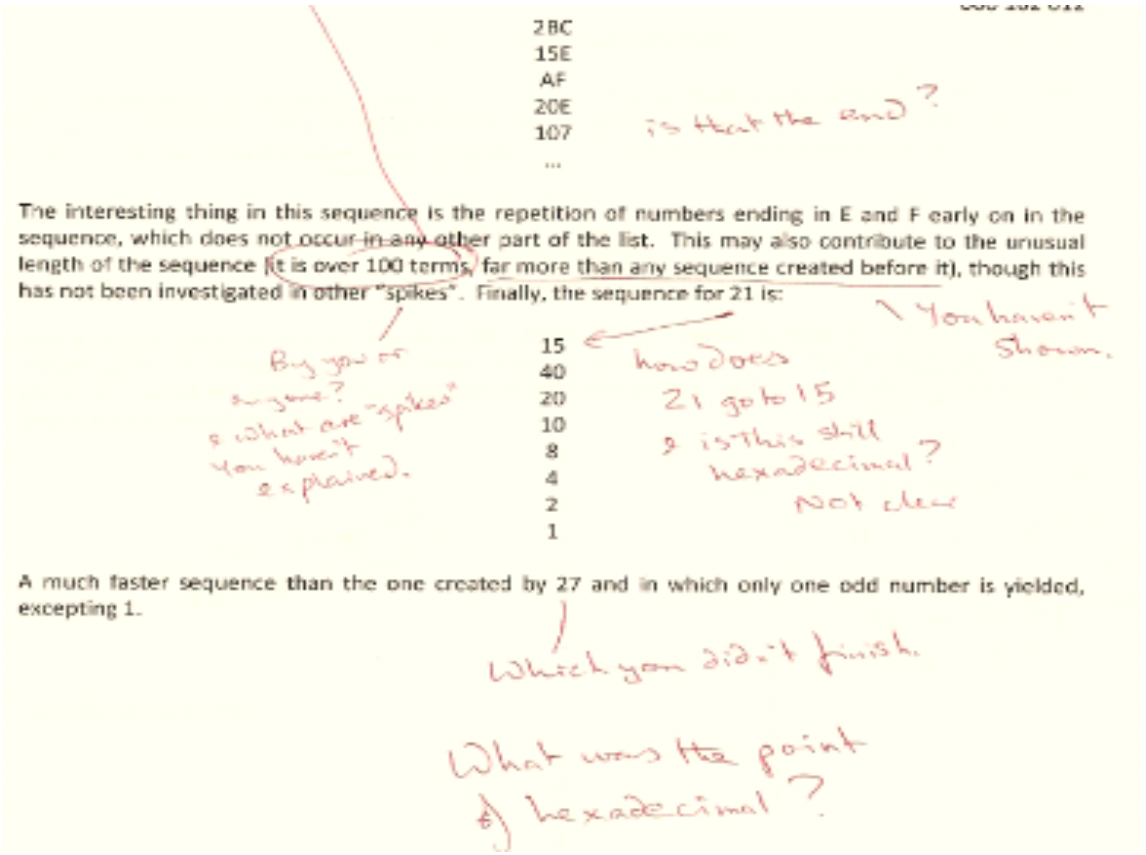
IBI = [(-2x64) – (8x10)]

IBI = -208

The result of the 2x2 determinant is even. I have tried using other different random even integers and their determinants are still even. — Prove it.

This really does suggest a lack of real understanding and if this were typical of the whole essay (it was) categories D to G would be quite low.

5. Make sure students take opportunities to do the Maths rather than just talk about it.



2BC
15E
AF
20E
107 — is that the end?
...

The interesting thing in this sequence is the repetition of numbers ending in E and F early on in the sequence, which does not occur in any other part of the list. This may also contribute to the unusual length of the sequence (it is over 100 terms, far more than any sequence created before it), though this has not been investigated in other "spikes". Finally, the sequence for 21 is:

15
40
20
10
8
4
2
1

By you or anyone? What are "spikes" You haven't explained.

How does 21 go to 15 ? Is this still hexadecimal? Not clear

You haven't shown.

A much faster sequence than the one created by 27 and in which only one odd number is yielded, excepting 1.

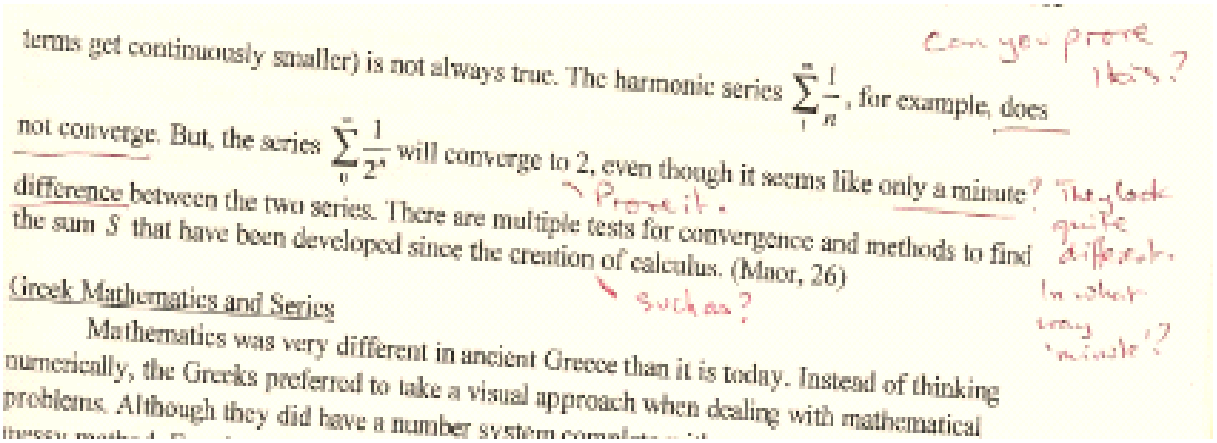
Which you didn't finish.

What was the point of hexadecimal?

This candidate decided to investigate the Collatz conjecture in hexadecimal (and binary) - both pointless, didn't explain his terms, didn't show what he

claimed and never really got to grips with the problem. It scored low, particularly on categories C to G and K.

6. **Don't allow the essay to quote results - actually do some Maths!**



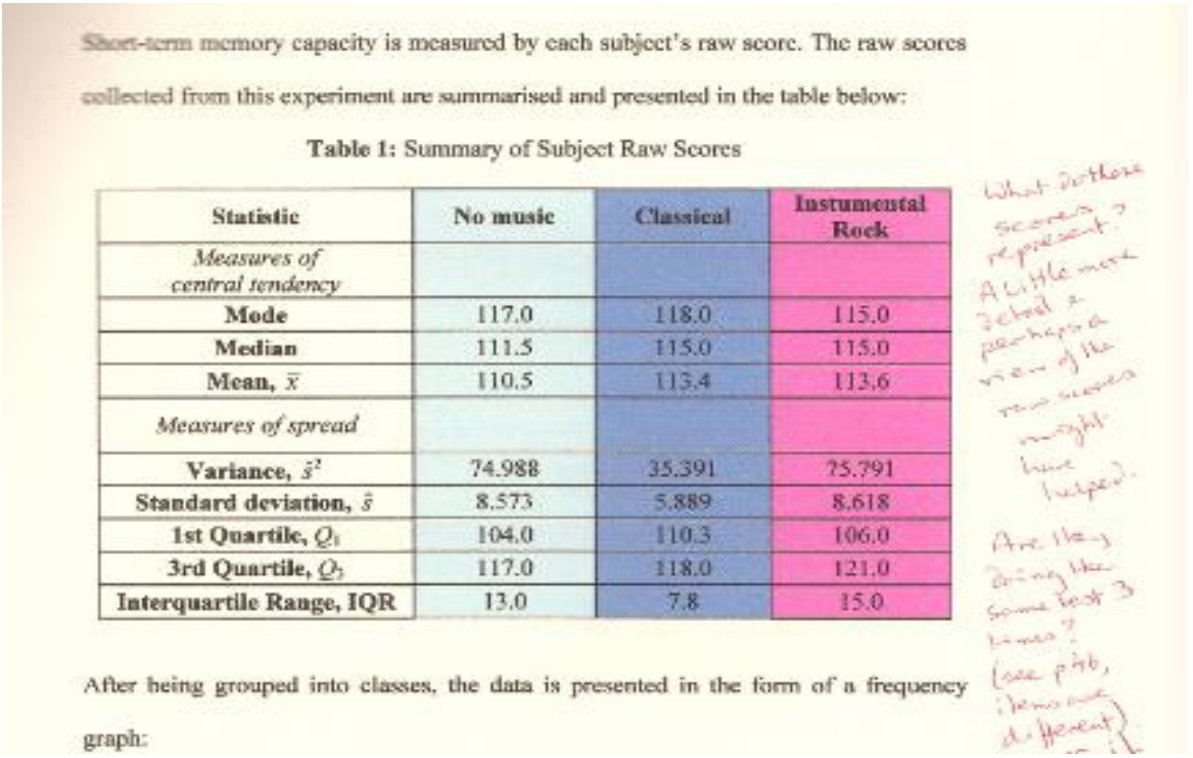
terms get continuously smaller) is not always true. The harmonic series $\sum_{1}^{\infty}\frac{1}{n}$, for example, does not converge. But, the series $\sum_{0}^{\infty}\frac{1}{2^n}$ will converge to 2, even though it seems like only a minute difference between the two series. There are multiple tests for convergence and methods to find the sum S that have been developed since the creation of calculus. (Maor, 26)

Greek Mathematics and Series

Mathematics was very different in ancient Greece than it is today. Instead of thinking numerically, the Greeks preferred to take a visual approach when dealing with mathematical problems. Although they did have a number system

This essay on series didn't actually do any maths, it just talked about it and quoted results, even though some of them aren't difficult to prove, e.g. the sum above does converge to 2 and is an infinite GP. This on scored poorly, it wasn't well planned and the candidate did no mathematics at all, fortunately it was well presented and the abstract and conclusion were good. It didn't score well elsewhere.

7. **In a statistical project, start with the raw data (put in the appendix if too big) and decide on summary statistics, charts, possible distributions and tests based on a consideration of that data.**



Short-term memory capacity is measured by each subject's raw score. The raw scores collected from this experiment are summarised and presented in the table below:

Table 1: Summary of Subject Raw Scores

Statistic	No music	Classical	Instrumental Rock
Measures of central tendency			
Mode	117.0	118.0	115.0
Median	111.5	115.0	115.0
Mean, $\bar{x}$	110.5	113.4	113.6
Measures of spread			
Variance, $\tilde{s}^2$	74.988	35.391	75.791
Standard deviation, $\tilde{s}$	8.573	5.889	8.618
1st Quartile, Q_1	104.0	110.3	106.0
3rd Quartile, Q_3	117.0	118.0	121.0
Interquartile Range, IQR	13.0	7.8	15.0

After being grouped into classes, the data is presented in the form of a frequency graph:

What do these scores represent? A little more detail perhaps a mean of the raw scores might have helped.

Are they doing the same test 3 times? (see p4b, remove different)

We didn't get to see the raw data, or what test was actually carried out, so E, F, G particularly might suffer. The candidate didn't make clear what he was doing. Its worth remembering that a student could do a Statistics essay as a Maths EE and could get the data from any source, so an interest in music (in this case) or sport, politics - anything - can generate data for a Statistical Maths EE. They don't always score highly - tend to be rather repetitive - but if point 1 above is remembered, it is still possible to get a good grade on something that a candidate is often really interested in.

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